UNIVERSIDAD DE GRANADA

Departamento de Estadística e Investigación Operativa

Seminario 4 (SPA Series): Fractional stochastic processes

22/03/2025

Seminario de la profesora **Enrica Pirozzi** (Dipartimento di afferenza: Dipartimento di Matematica e Fisica (DMF). Università della Campania Luigi Vanvitelli) (



https://www.matematicaefisica.unicampania.it/dipartimento/docenti-csa?MATRICOLA=904686) en la **sala de conferencias del IMAG**. Dos sesiones, el **24 de marzo** de 2025, **de 9:30-11:00 y de 11:30 a 13:00.**

Part I: **Fractional stochastic processes for modeling some biological dynamics: theoretical setting, modeling approaches, numerical comparisons and simulations**

Motivated by the need to model some neurophysiological evidences not included in classical neuronal models, we construct stochastic models based on coupled fractional stochastic differential equations, with different fractional orders. Indeed, one of the main motivations is that to model neuronal dynamics on different time-scales. We give explicit expressions of the process representing the voltage variation in the neuronal membrane. Numerical evaluations of the average behaviors of involved processes are presented in order to put in evidence the features of the

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proposed models.

In order to refine the theoretical setting, we focus on Mittag–Leffler (ML) fractional integrals involved in the solution processes of a more general system of coupled fractional stochastic differential equations. We introduce the ML fractional stochastic process as a ML fractional stochastic integral with respect to a standard Brownian motion. We provide some representation formulas of solution processes in terms of Mittag–Leffler fractional integrals and processes. Computable expressions of the mean functions and of the covariances of such processes are specifically given. The application in neuronal modeling is provided, and all involved functions and processes are specifically determined. Numerical evaluations are carried out and some results are shown and discussed.

References

[1] Abundo, M.; Pirozzi, E. Fractionally integrated Gauss-Markov processes and applications. In Communications in Nonlinear Science and Numerical Simulation; Elsevier: Amsterdam, The Netherlands, 2021; Volume 101, p. 105862. ISSN 1007-5704.

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Part II: **Time-changed stochastic models and fractionally integrated processes to model the actin-myosin interaction and dwell times**

We propose two stochastic models for the interaction between the myosin head and the actin filament, the physio-chemical mechanism triggering the muscle contraction and at the moment not completely understood. We make use of the fractional calculus approach with the purpose to construct non-Markov processes for models with memory. A time-changed process and a fractionally integrated process are proposed for the two models. Each of these include memory effects in different way. We describe such features from a theoretical point of view and also with simulations of sample paths. Mean functions and covariances are provided taking into account

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constant and time-dependent tilting forces by which effects of external loads are included. The investigation of the dwell time of such phenomenon is carried out by means of density estimations of the first exit time (FET) of the processes from a strip: this mimics the times of the steps of the myosin head during the sliding movement outside a potential well due to the interaction with the actin. For the case of the time changed diffusion process we specialize an equation for the probability density function of the FET from a strip. The schemes of two simulation algorithms are provided and performed. Some numerical and simulation results are given and discussed.

References

[1] Leonenko N., Pirozzi .E., The time-changed stochastic approach and fractionally integrated processes to model the actin-myosin interaction and dwell times. Mathematical Biology and Engineering. (Submitted).

[2] Leonenko, N., Pirozzi, E. (2021). First passage times for some classes of fractional time-changed diffusions. Stochastic Analysis and Applications, 40(4), 735–763. https://doi.org/10.1080/07362994.2021.1953386.

[3] Kobayashi, K. Stochastic Calculus for a Time-Changed Semimartingale and the Associated Stochastic Differential Equations. J Theor Probab 24, 789–820 (2011). https://doi.org/10.1007/s10959-010-0320-9.

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