UNIVERSIDAD DE GRANADA

Departamento de
Estadística e
Investigación Operativa

Seminario 7 (SPA Series): Fractional hyperbolic diffusions on sphere with random data

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Seminario del profesor **Nikolai Leonenko** (Cardiff School of Mathematics. Cardiff University) (



https://mathsdemo.cf.ac.uk/maths/contactsandpeople/profiles/leonenkoN.html) en la **sala de conferencias del IMAG**. Dos sesiones, el **28 de marzo** de 2025, **de 9:30-11:00 y de 11:30 a 13:00.**

Fractional hyperbolic diffusions on sphere with random data

Part I. **Introduction to statistical analysis of Gaussian isotropic spherical random fields**

Spherical random fields are very useful for modelling some phenomena in areas such as earth sciences (like, for example, in geophysics and climatology and cosmology). In fact, the application of statistical methods in cosmology has become increasingly important due to the many experimental data obtained in recent years, and spherical random fields are of particular interest regarding the analysis of Cosmic Microwave

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Background (CMB) radiation discovered by the astronomers Arno Penzias and Robert Wilson in 1964.

As well-known, the CMB is a spatially isotropic radiation field spread throughout the visible universe, originated around 14 billion years ago, and it is the main source of information we have about the evolution of the universe. The CMB radiation can be mathematically modelled as an isotropic spherical random field for which there is a spectral representation by means of spherical harmonics.

We consider statistical analysis of Gaussian isotropic spherical random fields, when only one observation of the field is available. The characteristic function of these empirical covariances is given in terms of cumulants. It turns out that our estimator follows a Rosenblatt type distribution [1,2,3]. Our goal is to estimate the covariance function of the random field, given a single observation at each point of the discretized sphere. We present a methodology for handling the problem of cosmic variance in this framework.

This is joint results with M.Taqqu (Boston University, USA) and G.Terdik (Debrecen University, Hungary). Some results used in the talk are obtained jointly with M.D.Ruiz-Medina (Granada University, Spain).

Part II. **Fractional hyperbolic diffusions on sphere with random data**

Our next objective is to study the fundamental solutions to fractional hyperbolic diffusion equation in the time variable using the Caputo derivative, and its properties. The exact solutions of the fractional hyperbolic diffusion equation with random data are derived in terms of series expansions of isotropic in space random fields on the unit sphere. The results are illustrated through motivating numerical examples of interest in several applied fields.

This is joint results with A. Olenko (La Trope University, Melbourne, Australia) and J.Vaz (UNICAMP, Brazil) [4,5]

References

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[5] Leonenko, N., Olenko, A. and Vaz, J. (2024) On fractional spherically restricted hyperbolic diffusion random field, Communications in Nonlinear Science and Numerical Simulation, 131,107866

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